A Combinatorial Analysis of Tic-Tac-Toe and The Theoretical Advantage of Playing First

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***Abstract*— The game of tic-tac-toe is a well-known paper-and-pencil game that is played by two players. It is a simple game, yet when viewed purely by numbers, it can get interesting. Combinatorics can be used to analyze the game and gain a better understanding of the metrics of tic-tac-toe. After analysis, it is concluded that there are a total of 19 683 possible states of a tic-tac-toe game board, without minding the rules of the game. When considering rules and limitations, the number of reachable game states reduces to 5478. There are 255 168 possible, playable games of tic-tac-toe, where 51% of them are won by the player who plays first, with a clear advantage for them in other metrics too.**

***Keywords*—Tic-Tac-Toe, Combinatorics, Board Game, Permutations, Combinations.**

I. Introduction

Known as 'tic-tac-toe', 'noughts and crosses', and 'Xs and Os' amongst other names, this simple two player game is played throughout the world even across different cultures. The origins of this paper-and-pencil game, from now on referred to as tic-tac-toe, are not certain, although variations of it have been traced back to as far as 1300 BC in ancient Egypt, with three-in-a-row type game boards found on etched on to roof tiles. The game developed from variations such as *terni lapilli* ('three pebbles at a time') which was found to be played in the Roman Empire, although the rules were quite different then. As history progressed, the modern, specifically English, names started appearing around the 1800s, and so did the modern rules too. The simplicity of setting up and playing tic-tac-toe is probably the reason this game has withstood the test of time and thrived to become a classic game amongst the people.

If one were familiar with tic-tac-toe, one would quickly notice that it is strategically a simple game. If both players play the optimal strategy, the game will always lead to a draw. There are several interpretations of the optimal strategy. One of them is laid out by Crowley and Siegler in 1993, which follows 8 rules that are ordered specifically in a hierarchy:

1. Win: Complete a three-in-a-row.
2. Block: Block the opponent from a three-in-a-row.
3. Fork: Create two possible three-in-a-rows.
4. Block Fork: Block opponent from making a fork.
5. Center: Play center if blank
6. Opposite Corner: Play the opposite corner of opponent.
7. Corner: Play corner if blank
8. Side: Play side if blank

A player only has to follow these rules in the exact order to achieve an expert play. This will result in in either a guaranteed win if the opposing player does not follow the optimal play and blunders, or a draw if the opposing player is optimal. This only counts if the player is starting first. If the second player plays optimally, the outcome would always be a draw. Of course, there are more intuitive strategies to follow, but these ordered rules allow an easier way to make an algorithm that plays the optimal tic-tac-toe.

In game theory, a game like tic-tac-toe that always results in a draw is called a futile game. So, why analyze a game so trivial and futile? As simple as the game might be, there are still aspects of it that are of interest for further analysis. This paper will view tic-tac-toe from a combinatorial perspective, steering away from strategic and algorithmic approaches, and towards pure numbers and combinatorics. The analysis will answer questions such as, among others: How many possible game board states are there? How many reachable game states are there? How many possible different games of tic-tac-toe can be played? How significant is the advantage of starting first when viewed combinatorically? These questions may seem trivial but are in fact quite a bit more complex than they seem. The simple rules limit the possible combinations of game states and result in interesting outcomes.

II. Theoretical Framework

A. Tic-Tac-Toe

For completeness' sake, the rules of tic-tac-toe will be laid out to assist and validate our analysis. The game board is often constructed by drawing 4 lines, two of them parallel vertically, and the other two horizontally, essentially creating a 3x3 grid in the simplest way. Of course, it can be made differently as long as it has 3 rows and 3 columns. Each of the two players will be assigned a mark, usually an X or an O, and take turns filling up the grid with their respective marks. A player can only play on empty slots.

A player wins if they are the first to arrange three of their marks in a row. This can be done horizontally, vertically, or diagonally. The game stops after a win. If the board is completely filled without anyone arranging three marks in a row, then the game results in a draw.

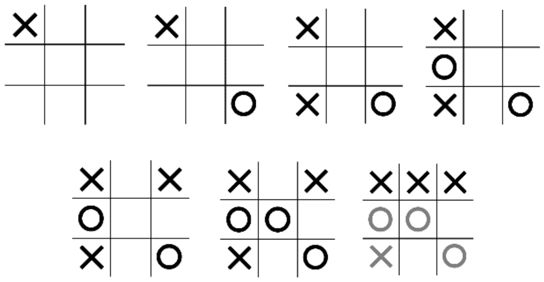

FIGURE I. AN AVERAGE GAME OF TIC-TAC-TOE

There is no defined rule for determining which player starts first, but for this analysis, it will be assumed that the player assigned with the mark 'X' always starts first. Therefore, there will be at most 5 X's and 4 O's when the board is full. This will be very important for our calculations and will be elaborated further in the following sections.

B. Combinatorics

Combinatorics is a branch of mathematics that deals with counting the number of arrangements of objects or finite structures without necessarily enumerating all possible combinations or permutations by hand. In short, combinatorics can be summed up by the question "How do you count without counting?". There are a lot of topics that are discussed under combinatorics, but two of the simpler concepts in combinatorics are the sum rule and the product rule, which will prove to be helpful in analyzing our case of tic-tac-toe.

The sum rule is quite intuitive and states that if there are n(A) ways to do A and n(B) ways to do B such that the phenomena A and B are distinct and not related to one another, then there are n(A) + n(B) ways to do A *or* B. A simple example: John wants to go to a neighboring city on the weekend. There are 3 bus services and 2 train services leading to that city. John can choose either going by bus or train. Thus, John has 3 + 2 = 5 ways to get to the neighboring city.

The product rule is also intuitive. It states that if there are n(A) ways to do A and n(B) ways to do B such that the phenomena A and B are distinct and not related to one another, then there are n(A) × n(B) ways to do A *and* B. Notice the difference between the two, one represents *or*, and the other *and.* An example of this rule would go as such: Frank is in California, and he wants to fly to India. There are 3 flights from California to France, and 2 flights from France to India. Frank is fine with transiting through France. Thus, Frank has 3 × 2 = 6 ways to fly to India from California.

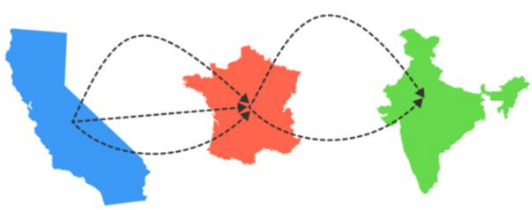

FIGURE II. FLIGHTS AVAILABLE TO FRANK. SOURCE: HTTPS://BRILLIANT.ORG/WIKI/RULE-OF-SUM-AND-RULE-OF-PRODUCT-PROBLEM-SOLVING/

C. Permutations

Another aspect of combinatorics is permutation. Permutations can be defined as the number of arrangements of a selection of objects, *with regards to their order*. If we take a row of n objects, there would be n slots where we could put and rearrange these objects. Starting from the first slot, there would be n possible objects to put there. The second slot can have $n-1$ possible objects. The next slot has $n-2$, and so on and so forth. Applying the product rule, it can be said that the number of ways to arrange a row of n objects can be calculated as such:

$$P(n,n) = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1 = n! \quad (1)$$

Permutations can also be more generalized. A permutation of n objects into r slots, or taking r objects from a set of n different objects, with $r \leq n$ and written as $P(n,r)$ can be calculated with the following formula:

$$P(n,r) = n(n-1)\ldots(n-r+1) = \frac{n!}{(n-r)!} \quad (2)$$

Another possibility is the permutation of n objects into n slots, but with some objects that are the same, or indistinguishable, from a number of other objects. If there were n_1 of a type of objects, n_2 of another type of objects, and so on to n_k, then (1) can be generalized into:

$$P(n; n_1, n_2, \ldots, n_k) = \frac{n!}{n_1! n_2! \ldots n_k!} \quad (3)$$

D. Combinations

Similar yet different than permutations, combinations are also an important part of combinatorics, thus the name. Akin to permutations, combinations are the number of arrangements of a selection of objects, *without regards to their order*. Since order does not matter, the number of combinations is always lower than the number of permutations. A combination of n objects into r slots, written as $C(n,r)$ can be formulated as such:

$$C(n,r) = \frac{n(n-1)\ldots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!} \quad (4)$$

What if there are multiple identical items? Combinations can be further generalized in this case. In the analogy of objects and slots, identical categorization can be interpreted as the slots allowing multiple objects inside of them. If there are r objects to be put inside n slots (note: the reverse notation as before), then the number of ways those objects can be put inside the slots is calculated with the following notation:

$$C(n+r-1, r) = C(n+r-1, n-1) \quad (5)$$

Both of these calculations will result in the same number of combinations when put into equation (4). If there is a limit of how many objects can be put into a certain slot, then one can use the sum rule to calculate each possibility, or reduce the number of objects available.

III. ANALYSIS

A. Board States

The first analysis will be on the number of possible states the tic-tac-toe game board can be in. A board state is defined as the state of a tic-tac-toe board, in which each slot of the 3x3 board can be assigned to three different states, an X, an O, or possibly also an empty slot. In this first analysis, the rules of tic-tac-toe and the limitations thereof will be ignored, and focus will be put on purely the number of states the board can be in. With 9 slots available on the board and each slot having 3 possible states, we can use the product rule of combinatorics to calculate the number of states to be:

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^9 = 19683. \quad (6)$$

Thus, there are 19 683 ways to put in or not put in the marks X and O on a tic-tac-toe game board. Of course, a portion of these states will not be possible during a real game of tic-tac-toe, or in other words: unreachable. Therefore, further analysis is required.

B. Reachable Game States

A possibly more useful metric would be the number of game board states that *are* reachable in a tic-tac-toe game. This analysis is considerably more complex than the first, since we have to obey the rules of tic-tac-toe and consider them in the calculations. We shall define a couple of variables first. The number of X marks, O marks, and empty slots shall be defined as $n_x, n_o,$ *and* n_e consecutively.

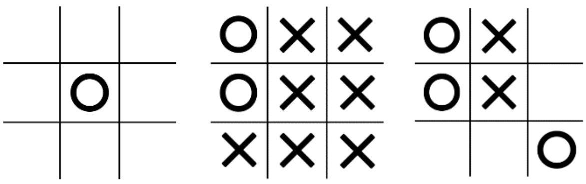

FIGURE III. EXAMPLES OF UNREACHABLE GAME STATES

The first rule we must consider is not necessarily a rule, but the nature of the game. Two players take turns in placing their marks on the board. With the assumption that the player with the mark 'X' starts first, that means that the number of X's, O's, and empty slots follow:

$$n_e + n_x + n_o = 9 \quad (7)$$
$$n_x = n_o \ or \ n_x = n_o + 1 \quad (8)$$

With this in mind, we can define every possible turn of the game in terms of these variables. For example, the start of the game would be $n_x = 0, n_o = 0, n_e = 9$, after the first turn by player X, the turn would be defined as $n_x = 1, n_o = 0, n_e = 8$, and so on for every turn until $n_x = 5, n_o = 4, n_e = 0$. To calculate every possible state at every turn, we can use the permutation in (3), with 9 slots available on the board and n_x, n_o, n_e numbers of identical marks, i.e., objects. Thus, the number of possible permutations at every turn is as follows:

$$P(9; n_x, n_o, n_e) = \frac{9!}{n_x! n_o! n_e!} \quad (9)$$

Start of the game ($n_x = 0, n_o = 0, n_e = 9$):
$$P(9; 0, 0, 9) = \frac{9!}{0!0!9!} = 1 \quad (10)$$

After turn 1 ($n_x = 1, n_o = 0, n_e = 8$):
$$P(9; 1, 0, 8) = \frac{9!}{1!0!8!} = 9 \quad (11)$$

After turn 2 ($n_x = 1, n_o = 1, n_e = 7$):
$$P(9; 1, 1, 7) = \frac{9!}{1!1!7!} = 72 \quad (12)$$

After turn 3 ($n_x = 2, n_o = 1, n_e = 6$):
$$P(9; 2, 1, 6) = \frac{9!}{2!1!6!} = 252 \quad (13)$$

After turn 4 ($n_x = 2, n_o = 2, n_e = 5$):
$$P(9; 2, 2, 5) = \frac{9!}{2!2!5!} = 756 \quad (14)$$

After turn 5 ($n_x = 3, n_o = 2, n_e = 4$):
$$P(9; 3, 2, 4) = \frac{9!}{3!2!4!} = 1260 \quad (15)$$

After turn 6 ($n_x = 3, n_o = 3, n_e = 3$):
$$P(9; 3, 3, 3) = \frac{9!}{3!3!3!} = 1680 \quad (16)$$

After turn 7 ($n_x = 4, n_o = 3, n_e = 2$):
$$P(9; 4, 3, 2) = \frac{9!}{4!3!2!} = 1260 \quad (17)$$

After turn 8 ($n_x = 4, n_o = 4, n_e = 1$):
$$P(9; 4, 4, 1) = \frac{9!}{4!4!1!} = 630 \quad (18)$$

After turn 9 ($n_x = 5, n_o = 4, n_e = 0$):
$$P(9; 5, 4, 0) = \frac{9!}{5!4!0!} = 126 \quad (19)$$

Now, games states at every reachable turn of the game can be calculated using the sum rule:

$$1 + 9 + 72 + 252 + 756 + 1260 + 1680 + 1260 + 630 + 126 = 6046 \quad (20)$$

The analysis so far implies that there are 6046 possible, reachable game states. This may seem like the end of this analysis, but there is one other thing we have to consider. Once a three-in-a-row is made, the game stops. Because of this, there are certain states containing three-in-a-row formations that are unreachable, mostly ones that are 1 turn too much from a winning state. To count these states, we must define all winning three-in-a-row formations.

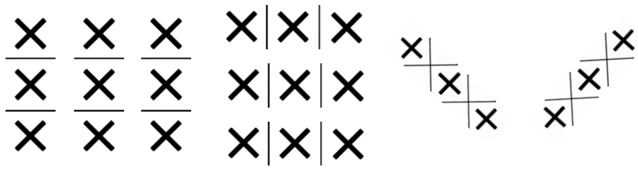

FIGURE IV. ALL POSSIBLE THREE-IN-A-ROW FORMATIONS FOR X

There are 8 winning formations for a given mark, with 3 vertical formations in every column, 3 horizontal formations in every row, and 2 diagonal formations in the diagonals. The remaining 6 slots beside the formation can be filled with marks, abiding the rules in (7) and (8). Now, if one does a little bit of trial and error, one can find that there are 4 cases in which a winning state is unreachable. A winning state for X but with the same amount (3) of O's in the remaining slots, a winning state for X but with 3 O's and 1 other X in the remaining slots, a winning state for O but with 4 X's, and the last case is a winning state for O but with 5 X's and one other O in the remaining slots. If we multiply each case with every possible three-in-a-row formation and add them up using the sum rule, we can count the amount of unreachable winning states:

$$8 \times P(6; 0, 3, 3) = 8 \times \frac{6!}{0!3!3!} = 8 \times 20 = 160 \quad (21)$$
$$8 \times P(6; 1, 4, 1) = 8 \times \frac{6!}{1!4!1!} = 8 \times 30 = 240 \quad (22)$$
$$8 \times P(6; 4, 0, 2) = 8 \times \frac{6!}{4!0!2!} = 8 \times 15 = 120 \quad (23)$$
$$8 \times P(6; 5, 1, 0) = 8 \times \frac{6!}{5!1!0!} = 8 \times 6 = 48 \quad (24)$$
$$160 + 240 + 120 + 48 = 568 \quad (25)$$

Now, we can subtract these seemingly reachable yet unreachable states from the previous number of states calculated in (20):

$$6046 - 568 = 5478 \quad (26)$$

With this, we have removed the unreachable states and finally obtained the number of possible and reachable game states in a game of tic-tac-toe, that is 5478 game states.

C. Playable Games

Another very interesting metric is the number of tic-tac-toe games that can be played. This is very different than the previous analysis on game states, since a game of tic-tac-toe consists of a series of turns that end in a win, loss, or draw. This number will be significantly higher. If we were to approach this problem naively, we would say that there are 9 possible ways of placing the first mark X in the first turn, 8 possible ways of placing an O mark in the second turn, and so on until the board is filled:

$$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 9! = 362880 \quad (27)$$

So, there are 362 880 ways to fill up the board if two players take turns. But notice how this does not fit the definition of a game, that is a series of turns *that end in a win, loss, or draw* for a player. Many, if not most of the games in (27) go beyond a game that was already won by a player, or don't have sufficient turns to result in a win, loss, or draw at all, since it takes at least 5 turns to generate a winning state. So, how do we calculate the number of games playable? To do that, we have to count the number of games ending in every turn from turn 5, other words, we apply the sum rule for games ending on turn 5, games ending on turn 6, games ending on turn 7, games ending on turn 8, and games ending on turn 9. These games cannot contain games that have already ended.

On turn 5, the only player who can win is player with the mark X, since there would be 5 marks on the board and according to rules (7) and (8) that could only be 3 X's and 2 O's. There are 8 winning formations that X could form, and for each of those there would be $P(3,3)$ ways to put those three X's in the three slots. We can put the remaining 2 O's in the remaining 6 slots in $P(6; 0, 2, 4)$ forms, with $P(2, 2)$ ways to do so. With the product rule, the number of games ending on turn 5 is as such:

$$8 \times P(3,3) \times P(6; 0, 2, 4) \times P(2,2) = 8 \times 3! \times \frac{6!}{0!2!4!} \times 2! = 8 \times 6 \times 15 \times 2 = 1440 \quad (28)$$

On turn 6, it would seem both player X and player O have the possibility to have won the game, since there would be 3 X's and 3 O's. But one has to consider that a player can only win on their own turn, thus only player O could win on turn 6. From here on out, we can conclude that player X can win only on odd number of turns, and player O can win only on even number of turns. Same as before, there are 8 winning formations too for mark O, and for each of those there are $P(3, 3)$ ways to make them. There are 3 X's that we can put into the remaining 6 slots, so that's another $P(6; 3, 0, 3)$ with $P(3, 3)$ ways each. But these games cannot contain games where X has won in the previous turn and where O formed a winning formation after that. Since the diagonal winning formations do not allow another diagonal of the other mark, only the vertical and horizontal forms are considered. For each of the orientations, there are $P(3,3)$ ways X and O can have winning formations at once, and $P(3,3)$ ways to put the X marks in and $P(3,3)$ ways to put in the O marks. We subtract these invalid 6 turn games from the previous calculation as such, to obtain the number of games ending on turn 6:

$$\left(8 \times P(3,3) \times P(6; 3, 0, 3) \times P(3,3)\right) - 2 \times (P(3,3) \times P(3,3) \times P(3,3)) = \left(8 \times 3! \times \frac{6!}{3!0!3!} \times 3!\right) - 2 \times (3! \times 3! \times 3!) = 5328 \quad (29)$$

On turn 7, X is the only possible winner. There are 4 X's and 3 O's on the board, so the calculation for winning formations of X is a little bit more complex. There are 8 three-in-a-rows as usual, but there would be not $P(4,4)$ ways to put in the X marks. Since it's the 7th turn, the 4th X cannot be placed outside of the three-in-a-row formation since it would make the game invalid. There are then 3 ways to put in the 4th X and $P(3,3)$ ways to put in the first 3 X's in the other slots. This then is times the ways you could put the lone X mark in the other 6 slots, so that's another $P(6; 1,0,5)$. For the O marks, there are 5 slots left, that can be counted with $P(5; 0,3,2)$ with $P(3,3)$ ways to do so. As on turn 6, there are games that are invalid because they contain winning games from the previous turn. The unplayable games contain both X and O winning formations, one each, but only combinations of either two horizontal or two vertical forms. The calculation is similar to turn 6, but without forgetting the lone X mark that will reside in one of the three remaining slots, thus $P(3; 1,0,2)$, and that the 4th X must be in the three-in-a-row formation, like mentioned before. Then the number of games ending on turn 7 would be:

$$\left(8 \times 3 \times P(3,3) \times P(6;1,0,5) \times P(5;0,3,2) \times P(3,3)\right) - 2 \times (P(3,3) \times 3 \times P(3,3) \times P(3,3) \times P(3;1,0,2)) = \left(8 \times 3 \times 3! \times \frac{6!}{1!0!5!} \times \frac{5!}{3!2!} \times 3!\right) - 2 \times (3! \times 3 \times 3! \times 3! \times 3) = 47952 \qquad (30)$$

On turn 8, the player with mark O wins. Calculations will be similar to turn 7, but with O as marks in the winning formations. There are 4 X's and 4 O's. Again, we have 8 three-in-a-row patterns, and the last O must be put in the three-in-a-row. So, there are 3 ways to put the last O and $P(3,3)$ ways for the first three O's. Also, the O outside of the three-in-a-row has 6 possible slots, $P(6;1,0,5)$. The 4 X's can reside in the remaining 5 slots, with $P(5;4,0,1)$ permutations and $P(4,4)$ ways to put them in. For the invalid games, everything is similar to turns 6 and 7 except there is 1 spare O and 1 spare X, which will reside in the spare 3 slots, thus $P(3;1,1,1)$. The last O has to be in the three-in-a-row too.

$$\left(8 \times 3 \times P(3,3) \times P(6;1,0,5) \times P(5;4,0,1) \times P(4,4)\right) - 2 \times (P(3,3) \times 3 \times P(3,3) \times P(4,4) \times P(3;1,1,1)) = \left(8 \times 3 \times 3! \times \frac{6!}{1!0!5!} \times \frac{5!}{4!0!1!} \times 4!\right) - 2 \times (3! \times 3 \times 3! \times 4! \times 3!) = 72576 \qquad (31)$$

Turn 9 is the last turn of the game and can result either in a win for X or a draw. Calculating all the valid games that end at turn 9 is quite complex and there are a wide variety of possibilities. Instead, we can use the numbers from all the previous turns to calculate them. Since all games that are a continuation of the games ending in all of the previous turns are invalid, we can subtract them from the number of possible games from (27). After ending on turn 5, the remaining 4 slots can be filled to continue an invalid game, with $P(4,4)$ possibilities. After turn 6, there would be 3 slots, and so on. Thus, the number of games ending on turn 9 can be calculated as such:

$$9! - (1440 \times 4!) - (5328 \times 3!) - (47952 \times 2!) - (72576 \times 1!) = 127872 \qquad (32)$$

There are 127 872 games ending on turn 9. This number includes player X wins and draws. If we want to get numbers for the two different outcomes, we can calculate the number of draws possible in a game of tic-tac-toe then subtract it from the total number of games ending on turn 9. There are 3 distinct draw patterns in tic-tac-toe:

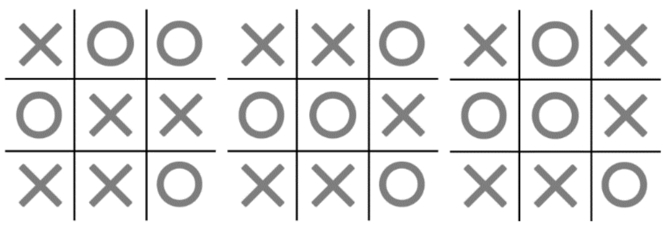

FIGURE V. 3 DISTINCT DRAW PATTERNS IN TIC-TAC-TOE

All games that end in a draw is ends in one of these patterns or a reflection or rotation of them. Thus, with reflections and rotations, there are 16 board states that are end states for games ending in a draw. Also calculate the fact that there are guaranteed 5 X's and 4 O's on the board, $P(5,5)$ ways to put in the X's and $P(4,4)$ ways to put in the O's. The number of games ending in a draw is as such:

$$16 \times P(5,5) \times P(4,4) = 16 \times 5! \times 4! = 46080 \qquad (33)$$

The number of game end on turn 9 with a win for X can now be calculated easily by subtracting from (32):

$$127872 - 46080 = 81792 \qquad (34)$$

Finally, the results of this particular analysis can be summed up into a table, with percentages relative to the number of playable games in total:

TABLE 1. PLAYABLE GAMES OF TIC-TAC-TOE

Game	Possibilities	Percentage
Win on turn 5 (X)	1440	0.56%
Win on turn 6 (O)	5328	2.09%
Win on turn 7 (X)	47952	18.79%
Win on turn 8 (O)	72576	28.44%
Win on turn 9 (X)	81792	32.05%
Draw	46080	18.06%
Total Playable Games	**255168**	**100%**

Thus, it can be concluded that there are 255 168 possible games of tic-tac-toe, with consideration to all the rules and limitations of the game. Although calculated differently, these numbers coincide with the findings by Henry Bottomley in 2001, which were used as a reference for this analysis.

D. Advantage of Playing First

Most people would know by intuition that playing on the first turn in tic-tac-toe is more advantageous, since one gets to choose where to play first, has more potential marks on the board, and so on. That is why tic-tac-toe is mostly played multiple times in a session, often the first to three points wins the session. But exactly how advantageous is playing first? By looking at our previous analysis, a new table can be made:

TABLE 2. PLAYABLE GAMES OF TIC-TAC-TOE

Game won by	Possibilities	Percentage
X	131184	51.4%
O	77904	30.5%
Draw	46080	18.1%
Total Playable Games	**255168**	**100%**

As can be seen in Table 2, if purely viewed by playable games and the combinatorics of it, the starting player, in this assumption the player with mark X, wins in approximately 51% of all playable games of tic-tac-toe, while the second player only wins about 30% of the time. The possible wins of the first player is 9 against 5 when compared to the wins of the second player. Meanwhile, these numbers also show that only 18% of games end in a draw.

Clearly, the starting player has an advantage even when viewed combinatorically. Another thing to consider is also the fact that the starting mark, placed anywhere on the board, will decrease the number of winning formations of the opposing mark, which could explain the difference. Of course, the number of marks possible on the board is also different, with a maximum of 5 X's and 4 O's if X starts the game. Thus, allowing X to form more possible winning states. Since the number of turns is also odd, there are more opportunities for the player on odd turns.

IV. Conclusion

In conclusion, combinatorics has proven itself to be useful for real applications, even as simple as a game of tic-tac-toe can have complex properties and can be solved. This analysis has provided answers for the questions asked in the introduction of this paper, that is 19 683 possible board states, 5478 reachable game states, and 255 168 playable games of tic-tac-toe. It has been proven that even when viewed with combinatorics, without considering strategies and algorithms, the starting player will always have the advantage, with 50% of possible games ending with their win. A starting player is in such an advantage because of several factors, such as having more marks to play, having more possible winning formations, and having more turns that allow them to win. An interesting finding was that even when most games of tic-tac-toe, in practice, end in a draw, only 18% of possible games of tic-tac-toe actually end in a draw.

It has to be said that much more applicative and conclusive analysis can be done if strategies and algorithms are applied, and that these findings differ greatly from actual gameplay of tic-tac-toe. When compared to algorithmic analysis, there are several differences in certain results of these calculations. Note also that there is a possibility that some of these calculations are proven to be false, since the writer is not particularly well-versed in this field. Even so, these findings may be considered when applying deeper analysis and/or applications on this game, especially when references of this topic are often not very descriptive.

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Declaration

I hereby declare this paper as my own writing, by my own hands, and not adapted, translated, nor plagiarized from any other existing works.

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